

[1] Read the following paragraphs and fill in the blanks from (A) to (J) with the appropriate formulas. Also, fill in the blanks (K) , (L) with suitable words.

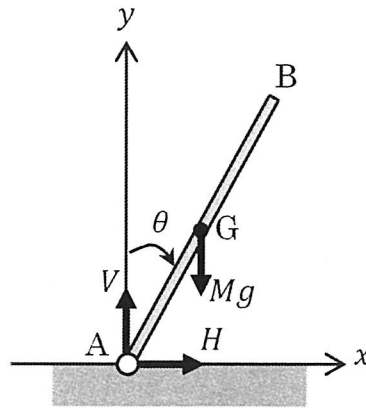


Figure 1

As shown in Figure 1, consider the in-plane motion of a rigid bar AB of length  $2a$  and mass  $M$  with uniform density, when its lower end A is fixed on a horizontal plane by a hinge (rotatable) and it falls from a nearly vertical state at rest. The motion of the bar can be defined by the angle of rotation  $\theta$  (clockwise positive), which is the tilt of the bar from the  $y$ -axis. Then, the rotation angle  $\theta$  in the initial nearly vertical state is assumed to be an infinitesimally small positive value. The only external force acting on the bar is due to the vertically downward gravity, and the gravitational acceleration is denoted as  $g$ . The horizontal reaction force  $H$  and the vertical reaction force  $V$  at point A can be determined in terms of the angle of rotation  $\theta$ .

First, the moment of inertia of the bar around point A is (A) (one also needs to show the derivation).

From an equation of motion for the rotation around point A,

$$\ddot{\theta} = \text{(B)} \quad (1)$$

and from the law of conservation of energy,

$$\dot{\theta}^2 = \text{(C)} \quad (2)$$

can be obtained. Next, let the coordinates of the position G, which is the center of gravity of the bar, be  $(x, y)$ , then the equations of motion in the horizontal and vertical directions for the center of gravity G are the following two equations, respectively.

$$\text{(D)} \quad (3)$$

$$\text{(E)} \quad (4)$$

Using the results of Equations (1) and (2), the horizontal reaction force  $H$  and the vertical reaction force

$V$  can be expressed in terms of the angle of rotation  $\theta$  as follows. Note that  $\ddot{\theta}$  and  $\dot{\theta}$  cannot be used.

$$H = \boxed{\hspace{2cm}} \text{ (F)} \quad (5)$$

$$V = \boxed{\hspace{2cm}} \text{ (G)} \quad (6)$$

From the above results, when the end of the bar (point B) reaches the horizontal plane, the horizontal reaction force  $H$  is  $\boxed{\hspace{2cm}} \text{ (H)}$  and the vertical reaction force  $V$  is  $\boxed{\hspace{2cm}} \text{ (I)}$ . When the angle of rotation  $\theta$  is  $\boxed{\hspace{2cm}} \text{ (J)}$ , the direction of the horizontal reaction force  $H$  changes from  $\boxed{\hspace{2cm}} \text{ (K)}$  to  $\boxed{\hspace{2cm}} \text{ (L)}$ .

[2] A satellite orbits in a circular orbit at a height  $h$  above the ground with a speed  $v$  (Figure 2). Assume that the Earth is a perfect sphere of mass  $M$  and radius  $R$  with uniform density (center of gravity is at the center of the Earth), the satellite has mass  $m$  and negligible size, and no force other than the force given by the law of universal gravitation acts between the Earth and the satellite. Fill the appropriate answers for (a) through (k) in the following blanks. Fill in  with an appropriate formula and  with a suitable phrase.

In terms of the universal constant of gravitation  $G$ , the universal force of gravitation  $F$  acting on the satellite, is expressed as follows.

$$F = \text{(a)}$$

Since  $F$  is the centripetal force for circular motion with speed  $v$ , the equation of motion is expressed as follows.

$$\text{(b)} = \text{(a)}$$

Then, the flight speed  $v$  of the satellite on the circular orbit can be expressed by a function of height  $h$ .

$$v = \text{(c)}$$

The lower the flight altitude is, the   $v$  becomes.

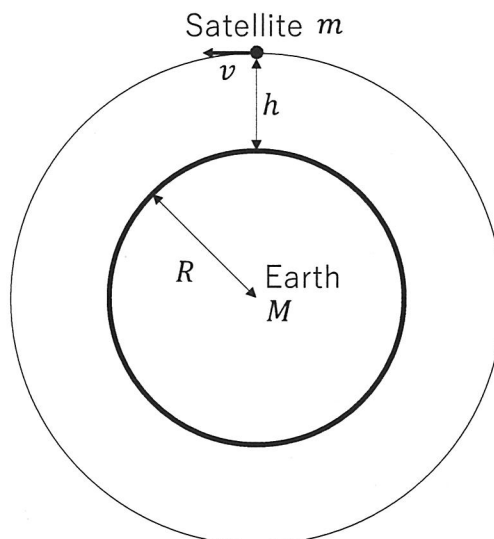


Figure 2

Next, consider the case when a satellite that has completed its mission is dropped to the ground. At the point A in Figure 3, the satellite is first decelerated to  $v_A$  to shift into an elliptical orbit, and when it reaches the point B on the opposite side of the Earth (altitude  $h/2$ ), it is decelerated to  $v_B$  and dropped half again to the point C on the Earth's surface just below the point A. The change in the satellite's mass due to deceleration is assumed to be sufficiently small to be negligible.

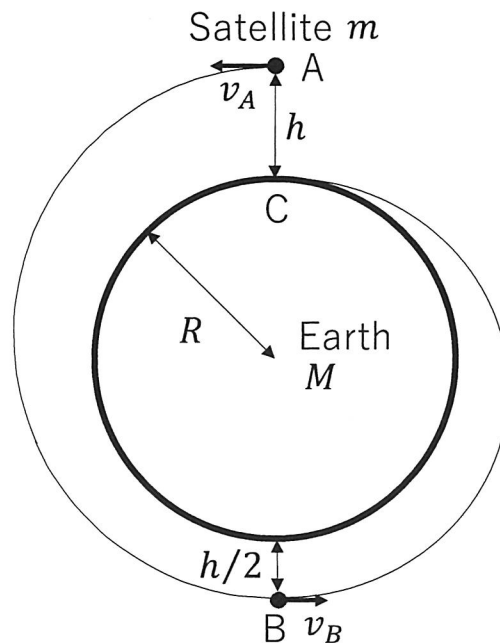


Figure 3

Generally, the major axis  $2a$  of the elliptical orbit is determined by the total energy  $E$  and is expressed by the following equation.

$$2a = \frac{GmM}{-E}$$

After the speed is reduced from  $v$  to  $v_A$  at the point A, the major axis of the elliptical orbit should be (e) to pass through the point B. Assuming that the potential energy is zero at infinite distance, the following equations can be established.

$$\frac{GmM}{\text{(e)}} = -E = - \left( \text{(f)} \right)$$

By rearranging this, the following is obtained.

$$v_A = \boxed{(g)}$$

Note that the speed  $v'_B$  when reaching the point B is expressed as follows from the Kepler's second law (a law of constant areal velocity).

$$v'_B = \boxed{(h)} v_A$$

So,  $v'_B$  is  $\boxed{(i)}$  than  $v_A$ .

After the speed is reduced from  $v'_B$  to  $v_B$  at the point B, the major axis of the elliptical orbit should be  $\boxed{(j)}$  to reach to the point C. Finally, the speed  $v_C$  when reaching the point C is expressed as follows.

$$v_C = \boxed{(k)}$$

Thus, by controlling the speed at the appropriate location, the satellite can be dropped to a safe point such as an ocean or a desert.