[1] As shown in Figure 1, consider rotation of a cylinder of homogeneous density with a mass M and a radius a of its section on a semi-cylinder with a radius b fixed on the horizontal plane. The center of the semi-cylinder is at O, the center of the cylinder is at P, and rotation angular velocity which is adopted as positive in clockwise direction is $\dot{\varphi}$, moving velocity of the center P is V. Forces F and N shown in the enlarged view in Figure 1 are in the directions parallel and perpendicular to the circumferential direction at the contact point, respectively. In addition, the axes of the cylinder and the semi-cylinder are parallel to each other, and no slip occurs between them. The circular constant is π, the gravitational acceleration is g, the cylinder is shorter than the semi-cylinder and both ends of the cylinder do not protrude from the semi-cylinder. Moreover, a force of air resistance is ignorable. Fill in the blanks (A) through (M) in the text below with appropriate equations or mathematical expressions for [ ], words for [ ], and a value for [ ].

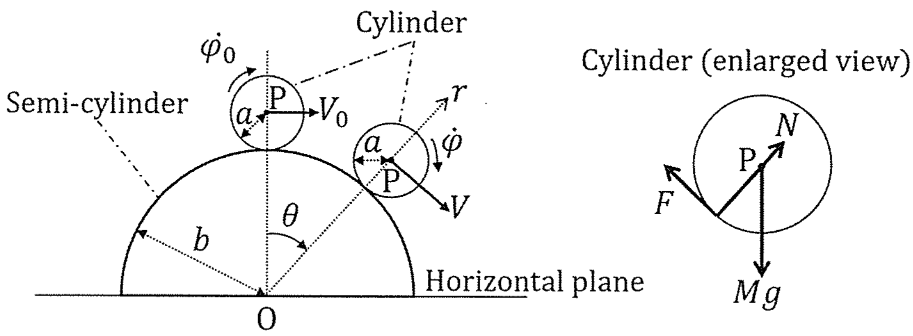

Figure 1 Rotational movement of a cylinder on a fixed semi-cylinder without slip

[1-1] The moment of inertia of the cylinder around its axis through central point P is [(A)]. In case of rotational movement of the cylinder on the fixed semi-cylinder without slip, the forces called [(B)], [(C)], [(D)] are acting on the cylinder.

[1-2] A polar coordinate system with the origin at the center O of the semi-cylinder is used to express the center P of the cylinder. OP and the vertical axis form the angle θ which is positive clockwise, and the cylinder never separates from the semi-cylinder (i.e., $\overline{\mathrm{OP}} = r = a + b$). In addition, assuming that the component of acceleration in radial direction is $-r\dot{\theta}^2$, the equations of motion for the center point $\mathrm{P}(r, \theta)$

of the cylinder in radial and circumferential directions of the semi-cylinder are (E) and (F), respectively. The equation of rotational motion around the center is (G).

[1-3] In the following, consider the motion of the cylinder after being released rightward from the initial location (angle of $\theta = 0$) at initial velocities of V_0 and $\dot{\varphi}_0$ as shown in Figure 1. Because slip never occurs between the cylinder and the semi-cylinder, the equation of energy conservation (H) expressed using $\dot{\varphi}$ and $\dot{\theta}$ between times of releasing (i.e., the angle of $\theta = 0$) and after releasing (i.e., an arbitrary angle of θ) is true.

[1-4] The radius of the semi-cylinder is equal to four times the radius of the cylinder, i.e., $b = 4a$. Considering the length of the arc that the cylinder rotated on the semi-cylinder during a short time Δt, the arc on the cylinder is equal to the arc on the semi-cylinder, it leads to (I) $= b\dot{\theta}\Delta t$ because no slip occurs between them. Utilizing this relation, the equations of motion in [1-2] and the equation of energy conservation in [1-3], an equation of force $N=$ (J) at an arbitrary angle of θ can be derived. The rotating cylinder separates from the semi-cylinder at the angle of $\theta_1 =$ (K), before arriving at the horizontal plane.

[1-5] The larger the initial velocity V_0 is, the (L) the angle of θ_1 is, where the cylinder separates from the semi-cylinder. Adopting the same relation of $b = 4a$ as that in [1-4], to satisfy a condition in which the cylinder rotates at least some distance, i.e., the cylinder does not separate immediately from the semi-cylinder, the initial velocity V_0 must be less than $\sqrt{\text{(M)}\, ag}$.

[2] As shown in Figure 2(a), a rectangular rigid body of uniform density and mass M is placed on a horizontal floor. The rigid body is hinged to the floor at point O. The length of the diagonal of the rigid body is L and the angle between the side and the diagonal is α. The moment of inertia of the rigid body around point O is $ML^2/3$. The angle of rotation θ of the rigid body around point O is positive clockwise. The rigid body does not move or rotate in the direction perpendicular to the surface of the paper. Ignore air resistance, friction, and other energy losses. The acceleration of gravity is g. The unit of angle is radian. Read [2-1] – [2-3] and fill in the blanks [(A)] to [(O)] with the appropriate mathematical expressions.

[2-1] When a horizontal rightward force F is applied to the gravity center of the rigid body, the rigid body rotates around point O, and the rigid body is balanced and at rest with a rotation of angle θ (> 0), as shown in Figure 2(b). In this case, the equation of equilibrium of the moment around point O is as follows.

[(A)] = 0

In the above equation, the angle θ at which F is zero is $\theta =$ [(B)].

[2-2] Consider that the rigid body is balanced and at rest with $\theta = \theta_0$ ($0 < \theta_0 <$ [(B)]) by a horizontal rightward force F. Then, by removing the force F, a rotational motion occurs in the rigid body.

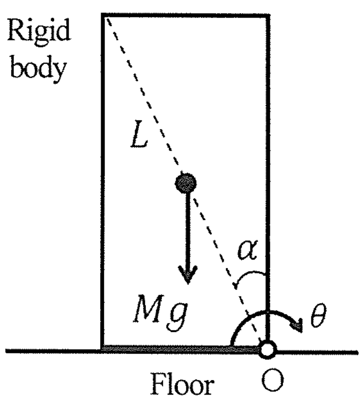

Figure 2(a)

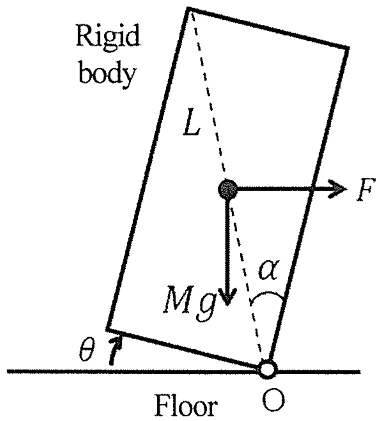

Figure 2(b)

If the force F is removed at the time $t = 0$, the equation of motion for the rotation is expressed as follows.

$$\frac{ML^2}{3}\ddot{\theta}(t) = \boxed{\text{(C)}} \tag{1}$$

We write $\theta(t)$ to make it clear that θ is a function of time t. Assuming that the approximations $\sin(\alpha - \theta(t)) \cong \alpha - \theta(t)$ and $\cos(\alpha - \theta(t)) \cong 1$ hold, and rearranging Eq. (1) for $\theta(t)$, we obtain

$$\ddot{\theta}(t) = \boxed{\text{(D)}}\left(\theta(t) + \boxed{\text{(E)}}\right) \tag{2}$$

Solving Eq. (2) under the initial conditions $\theta(0) = \theta_0$ and $\dot{\theta}(0) = 0$, $\theta(t)$ is expressed as

$$\theta(t) = \boxed{\text{(F)}} \tag{3}$$

From Eq. (3), the angle of rotation of the rigid body $\theta(t)$ decreases with time, and when it reaches zero, the rigid body collides with the floor. Since there is no energy loss due to the collision of the rigid body with the floor, the angle of rotation of the rigid body increases again, reaches a maximum value $\boxed{\text{(G)}}$, and then decreases again. Thus, the rigid body is in free vibration with period $\boxed{\text{(H)}}$.

[2-3] Initially, the rigid body is assumed to be at rest with a rotation angle of 0 as shown in Figure 2(a). A constant force F is applied to the center of gravity in the horizontal right direction from time 0 to t_0, and the rigid body is considered to rotate around point O. The equation of motion for this rotation is

$$\frac{ML^2}{3}\ddot{\theta}(t) = \boxed{\text{(I)}}. \tag{4}$$

Assuming that the approximations $\sin(\alpha - \theta(t)) \cong \alpha - \theta(t)$ and $\cos(\alpha - \theta(t)) \cong 1$ hold, and rearranging Eq. (4) for $\theta(t)$, we obtain,

$$\ddot{\theta}(t) = \boxed{\text{(J)}}\left(\theta(t) + \boxed{\text{(K)}}\right) \tag{5}$$

This indicates that the condition causing rotation of the rigid body is $F > \boxed{\text{(L)}}$. When $F > \boxed{\text{(L)}}$ holds, solving Eq. (5) under the initial conditions $\theta(0) = 0$ and $\dot{\theta}(0) = 0$, $\theta(t)$ is expressed as follows.

$$\theta(t) = \boxed{\text{(M)}} \tag{6}$$

Next, consider the duration of action t_0 of the force required for the rigid body to fall over, around point O. The work W done by force F until time t_0 is [(N)]. For the rigid body to fall over, it is necessary that $\theta >$ [(B)]. Since the increase in potential energy from the initial state when $\theta =$ [(B)] is $MgL(1 - \cos\alpha)/2$, the duration of action of the force required for the rigid body to fall over, around point O is $t_0 >$ [(O)].